INTERNAL STRESSES IN A BODY WITH COHERENT PARTICLES

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Knowledge of the internal stress field $\hat{\sigma}$ in a body containing an ensemble of coherent particles is necessary to the comprehension of many physical processes, for instance, hardening [1], spoilage of coherence [2], etc. The field $\hat{\sigma}$ is often evaluated by the formula $\hat{\sigma}=\sum_{i=1}^{N}\hat{S}_i$, where N is the quantity of particles in the body while $\hat{S}_i$ is the stress in a body of finite size produced by an individual i-th particle, i.e., under the conditions that in addition to it there are no other particles in the body (method 1) (see [3], e.g.). However, the equilibrium condition of the body as a whole turns out to be spoiled here. Up to now this circumstance has not been explained. Hence, the following formal method is used to evaluate $\hat{\sigma}$. The field $\hat{\sigma}$ is written in the form $\hat{\sigma}=\sum_{i=1}^{N}\hat{S}_i^{\infty}+\hat{\sigma}^{\mathrm{Im}}$, where $\hat{S}_i^{\infty}$ is the stress in an unbounded medium produced by an individual particle, while the homogeneous component $\hat{\sigma}^{\mathrm{Im}}$ is evaluated from the condition of equilibrium of the body as a whole. Ordinarily $\hat{\sigma}^{\mathrm{Im}}$ is treated as the stress due to imaginary forces [4, 5] (method 2). The stresses outside the particles computed by these methods agree, while the stresses within the particles are different. The application of the method 2 is legitimate when the elastic moduli is usually taken into account by renormalization of the deformation of the nonconformity between the particles and the body, the stress $\hat{\sigma}^{\mathrm{Im}}$ is here also considered homogeneous [4] (method 3). However, these assumptions require additional foundation.

It is shown in [6, 7] that the total elastic distortion in an unbounded medium due to a number of arbitrary defects* does not equal the sum of the distortions from isolated defects, i.e., the additivity principle for elastic distortions is not satisfied. The role of the effect of nonadditivity of the elastic stresses in a body of finite size that contains an ensemble of coherent particles is analyzed in this paper.

Let us consider a spherical body of radius R containing a statistically homogeneous distribution of spherical particles of the second phase. We limit ourselves to the case when the matrix and the phase are elastically isotropic with the Lamé coefficients μ_M, λ_M, and μ_p, respectively. Let all the particles have the identical radius r_0 and produce distortions determined in a dilatation center model with intensity ΔV_0 [8] (ordinarily the deformation of the nonconformity is $\varepsilon = \Delta V_0/(4\pi r_0^3) \ll 1$). The volume fraction of the second phase δ and N are connected by the relationship $\delta = N(r_0/R)^3$. We seek the field of elastic displacements in the following form [8]:

outside the particles

$$\mathbf{u}(\mathbf{r}) = A\sum_{i=1}^{N}(\mathbf{r}-\mathbf{r}_i)+B\sum_{i=1}^{N}\frac{(\mathbf{r}-\mathbf{r}_i)}{|\mathbf{r}-\mathbf{r}_i|^3}; \tag{1a}$$

within the i-th particle

$$\mathbf{u}(\mathbf{r}) = A_1(\mathbf{r}-\mathbf{r}_i),\ i=1,2,\ldots,N, \tag{1b}$$

where $\mathbf{r}_i$ is the radius-vector of the center of the i-th particle, and A, B and A_1 are constants determined from the boundary conditions, the continuity conditions for the normal stress on the interface between the phase and the matrice, and the magnitude of the jump in the displacement on the interface [8]. It is convenient to write the boundary condition in the form $\langle\sigma_{RR}\rangle = 0$, where σ_{RR} is the RR-th component of the stress tensor in a spherical co-

*The consideration in [6, 7] was performed in the linear elasticity theory approximation for the most common defect model, Somigliani dislocations.

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ordinate system with origin at the center of the body. The average is taken over the particle arrangement in the body. The condition of normal stress continuity on the interface between the i-th particle and the matrix is written conveniently for the quantity $\langle\sigma_{rr}\rangle$, where σ_{rr} is the rr-th component of the stress tensor in a spherical coordinate system with origin at the center of the i-th particle. It is convenient to evaluate the jump that the quantity $\langle u_r(\mathbf{r})\rangle$ undergoes during passage through the interface between the i-th particle and the matrix in the same coordinate system. For individual particles, $\langle u_r(\mathbf{r})\rangle$ undergoes a jump of magnitude εr_0 [8] on the interface. When an ensemble of particles is present in the body, it is necessary to take account of the nonadditivity of the elastic distortions. By using the method of [6, 7], the calculation of the jump in the function $\langle u_r(\mathbf{r})\rangle$ yields the value $[\varepsilon - A(N - 1)]r_0$. We write the equation to determine A, B and A_1. From the boundary conditions we obtain

$$A = \frac{4\mu_m B}{3K_m R^3}; \tag{2}$$

from the continuity of the normal stress on the interface

$$3K_m AN - 4\mu_m B/r_0^3 = 3K_p A_1; \tag{3}$$

from the condition for the jump in the displacement on the interface

$$A + B/r_0^3 - A_1 = [\varepsilon - (N-1)A], \tag{4}$$

where $K_m = (2\mu_m + 3\lambda_m)/3$, $K_p = (2\mu_p + 3\lambda_p)/3$ are the bulk moduli of the matrix and phase, respectively. The solution of the system (2)-(4) yields

$$A = \frac{4\mu_m}{I} K_p \varepsilon \left(\frac{r_0}{R}\right)^3, \quad B = \frac{3K_m}{I} K_p \varepsilon r_0^3, \quad A_1 = -\frac{4\mu_m}{I} K_m \varepsilon (1-\delta), \tag{5}$$

where $I = K_m(3K_p + 4\mu_m) - 4\mu_m(K_m - K_p)\delta$. The elastic dilatations θ_M, θ_p and the pressures p_M, p_p in the matrix and phase, respectively, are determined in the form

$$\theta_p = 3AN = \frac{4\mu_m}{I} 3K_p \varepsilon\delta, \quad p_m = -K_m \theta_m,$$
$$\theta_p = 3A_1 = -\frac{4\mu_m}{I} 3K_m \varepsilon (1-\delta), \quad p_p = -K_p \theta_p. \tag{6}$$

The mean pressure over the body volume is here

$$p_m(1-\delta) + p_p\delta = 0,$$

while the mean elastic dilatation over the body volume is

$$\theta_m(1-\delta) + \theta_p\delta = \frac{4\mu_m}{I} 3(K_p - K_m)\varepsilon\delta(1-\delta). \tag{7}$$

If N spheres of identical radius r_0 are imagined separated in a single-phase body, and then these sphere are replaced by particles of a second phase that cause dilatation distortions, then the body changes its volume. Such a replacement models the process of dissociation of a solid solution. The formula for evaluating the relative change in the volume V of a body has the form

$$\frac{\Delta V}{V} = 3\varepsilon\delta\left[1 - \frac{4\mu_m}{I}(K_m - K_p)(1-\delta)\right]. \tag{8}$$

Let us compare the results obtained with analogous relationships calculated earlier by known methods. In the case when the elastic moduli of the particles and the body are in agreement, (5)-(8) can be obtained by using method 2. For different moduli, the results obtained by the method 3 are not in agreement, since the passage to different moduli does not reduce the renormalization of ε. It is convenient to use the regularities (1), (5)-(8) in investigating the aging process.

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GENERAL MODEL OF SOMIGLIANI DISLOCATIONS

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1. Underlying the statistical description of plastic molding, substructure evolution, fracture,and other processes in real solids is the continual theory of defects (see [1-3], e.g.,). Among all the possible defects, dislocations and disclinations whose distributions can represent practically any substructures, occupy an important place. The examination of dislocations and disclinations as different defects is not always convenient and justified since they are Volterra dislocations (only of just a different kind). On the other hand, defects of the most general kind, Somigliani dislocations [2], can be the means for a single description of dislocations and disclinations. A step is made in this direction in [4] and a model is proposed for Somigliani dislocations, given by their basic plastic distortion fields β^p_{kl} and displacement velocities v^p_l. However, such Somigliani dislocations describes only the so-called dislocation model of defects [3]. This is completely adequate for a calculation of the dynamical elastic stress fields produced by defects, but certain disclination characteristics of the defect structure are not reflected here. The purpose of this paper is to obtain a general model of Somigliani defects which will equally take into account both the dislocation and the disclination characteristics of defects. As will be shown below, such a model should be a generalization of disclination (a rotational Volterra dislocation).

2. The usual (initial) definition of a Somigliani dislocation is formulated in terms of the total displacement fields u^T_l, which undergo arbitrarily changing jumps $[u^T_l]$ along S on the defect surface S [2]. In constructing the general model of a Somigliani dislocation, we proceed differently, namely, we give the definition of the model in terms of the basis plastic fields, as is done in [4].

We shall consider the general model of the Somigliani dislocation as a direct generalization of a disclination which is defined in the continual theory of defects by giving four basis plastic fields: ε^p_{kl} is the strain tensor, $\varkappa^p_{mq}$ is the bending-twisting tensor, v^p_l is the displacement velocity tensor, and w^p_q is the rotation velocity tensor [5, 6]. The expressions for the basis fields are obtained for an ordinary disclination by considering disclinations with a closed surface S(t) enclosing a volume V(t) where t is the time. The starting point is the expression for the total displacements $u^T_l(\mathbf{r}, t)$ within the volume V(t) [5]

$$u_l^T(\mathbf{r}, t) = \int\limits_V \delta(\mathbf{R}) \{b_l + \varepsilon_{lqr}\Omega_q (x'_r - x^0_r)\} \, dV', \tag{2.1}$$

where $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ is the difference between the radius-vectors of the observation and integration points, $\delta(\mathbf{r})$ is the three-dimensional Dirac delta function, b_l, Ω_q are the relative translation and rotation vectors of the edges of the slit S(t), ε_{lqr} is the unit antisymmetric-tensor, x_r are the Cartesian coordinates of the radius-vector $\mathbf{r}$, and x^0_r are coordinates of a point through which the axis of rotation passes. The basis fields are found by the following scheme [5, 6].

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